Ciphering 2018
Round 1.

1) Two sides of an isosceles triangle have lengths 3 and 8. What is the perimeter of the triangle?
Answer: 19.

2) What is the smallest positive integer k such that $\dfrac{k}{168}$ can be expressed as a terminating decimal?
(For example, $\frac{3}{4}$ can be expressed as a terminating decimal since $\frac{3}{4} = 0.75$.)
Answer: 21.

3) The sum of two real numbers is 12 and their product is 18. What is the sum of their reciprocals?
Answer: $\dfrac{2}{3}$.

4) How many positive integers have the property that the number obtained by deleting the units digit is equal to $1/14$ of the original number?
Answer: 2

5) Alice walks 50 meters in a straight line. She then turns 30 degrees to the left and walks another 50 meters, and then turns 30 degrees to the left again. She continues this pattern until she reaches the point where she started. How far did she walk?
Answer: 600 meters.

6) Let $x = (0.\overline{6}) \cdot (0.\overline{60})$. When x is written out as a decimal, what is the sum of the first 15 digits after the decimal point?
Answer: 32.

7) For what positive value of x is the equation $8^{x^2+1} = 4^{2x^2-1}$ true?
Answer: $\sqrt{5}$.

8) There are 4 children of different integer ages under 14. The product of their ages is 450. What is the sum of their ages?
Answer: 25.

Round 2.

1) If $a \odot b = a^b - b$, what is the value of $2 \odot (3 \odot 2)$?
Answer: 121.

2) If $2A756B$ is the smallest multiple of 18, written in base 10, what is the value of $B - A$?
Answer: 5.

3) The equation $x^3 + 4x^2 + x - 6 = 0$ has three integer solutions. What is the value of the smallest solution?
Answer: -3.

4) The director of a marching band asks the band members to line up in rows of four, but one is left over. Then she tries to line them up in rows of six, but three are left over. Finally, she tries to line them up in rows of seven, but four are left over. If the band has fewer than 100 members, how many members are there?
Answer: 81.

5) What is the units digit of $2^{3^{2018}}$?
Answer: 2.

6) Alexandra counted the sum of the angles of a convex polygon. She missed one of the angles and her result was $2018°$. What was the missed angle?
Answer $142°$.

7) Yuan wrote down four consecutive integers. He calculated the four possible totals made by taking three of the integers at a time. None of these totals was prime. What is the smallest integer Yuan could have written?
Answer: 7.

8) Suppose that 1365 is written as the product of three positive integers. What is the smallest possible value of the sum of these three integers?
Answer: 35.

Backup Problems

1) If a and b are the solutions of the equation $x^2 + 2017x + 2018 = 0$,

what is the value of $\frac{1}{a} + \frac{1}{b}$?

Answer: $-\frac{2017}{2018}$.

2) If $\sin x + \cos x = \frac{1}{2}$, what is the value of $\sin(2x)$?

Answer: $-\frac{3}{4}$.